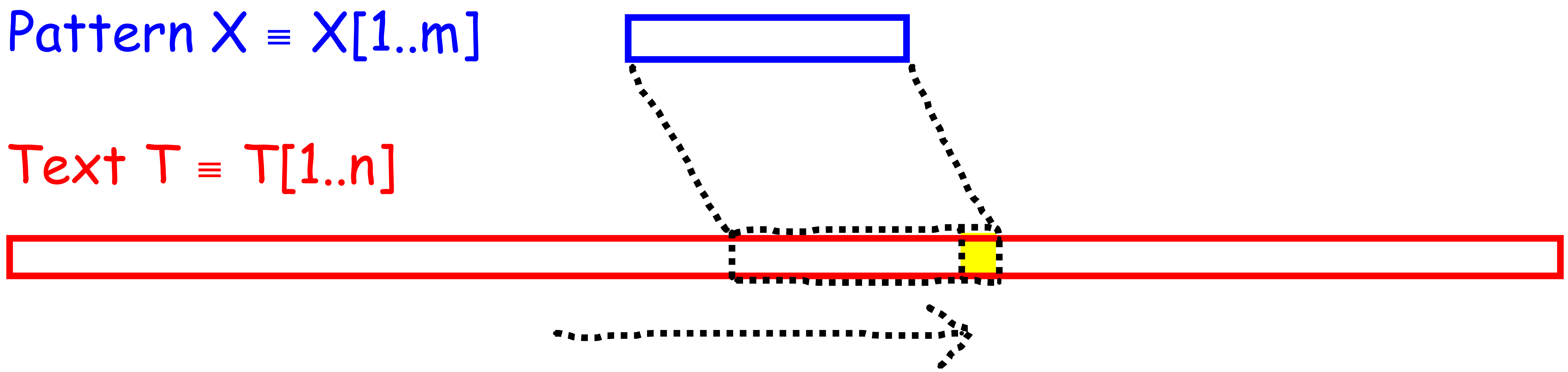
Simple Real-Time Constant-Space String Matching

Dany Breslauer, Roberto Grossi and Filippo Mignosi

Real-time string matching



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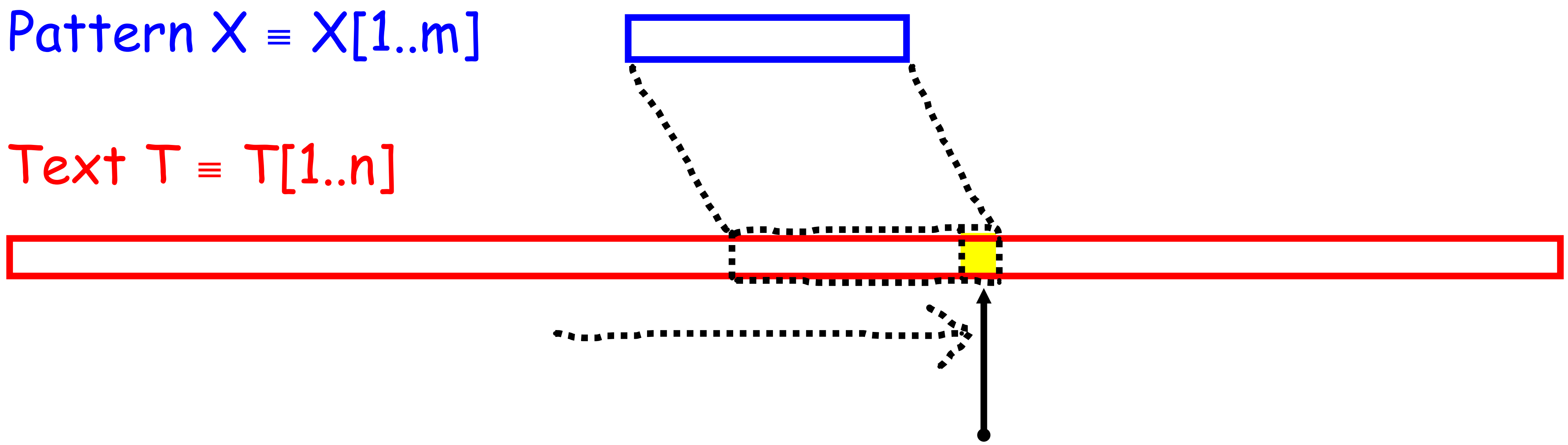

O(1) worst-case time to answer after reading the text symbol

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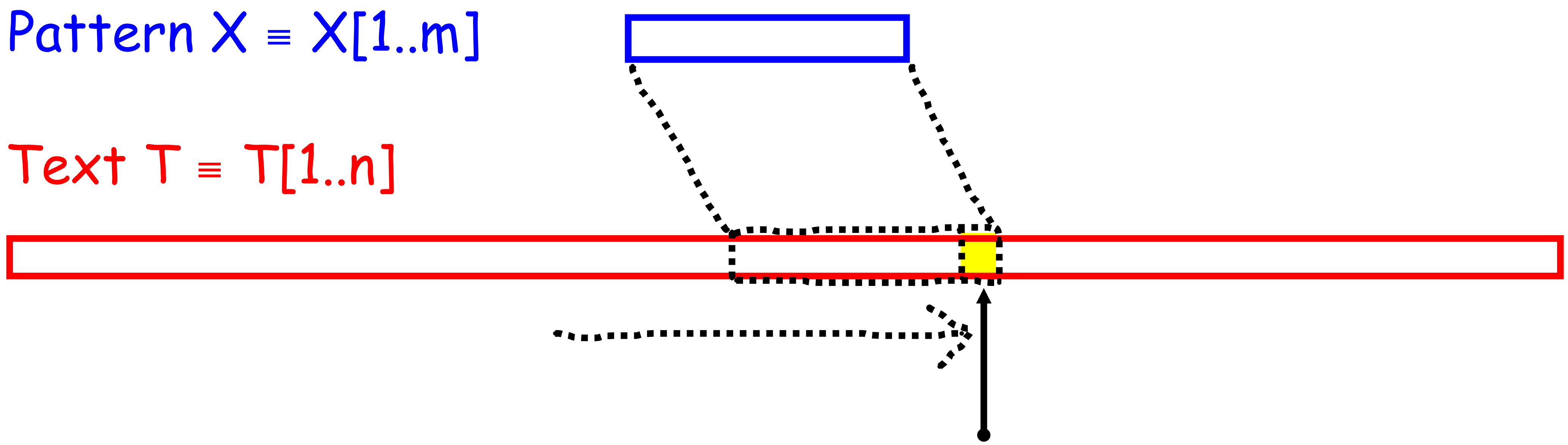


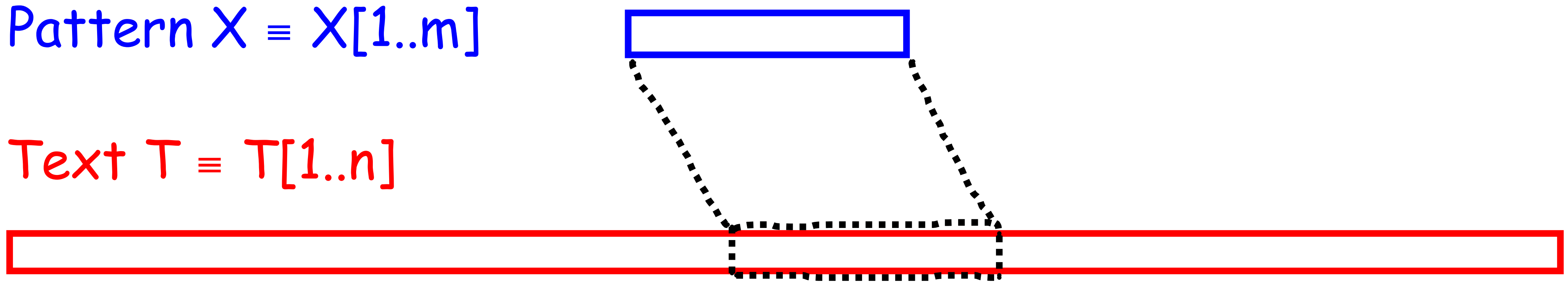
O(1) worst-case time to answer after reading the text symbol

Different from real-time streaming s.m., where X and T cannot be entirely stored!

Constant-space string matching

Pattern X ≡ X[1..m]

Text T ≡ T[1..n]



(a.k.a. in-place)

O(1) working space apart from that required by X and T

O(log n) bits

We propose a simple way to combine the two features

- Take a simple version of the constant-space Crochemore-Perrin (CP) algorithm

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- Make CP also real-time by running **two** instances simultaneously

Some related work

- Galil '81: real-time string matching
- Galil, Seiferas '83: constant space
- Karp, Rabin '87: randomized constant space real-time
- Crochemore, Perrin '91: constant space
- Gasieniec, Plandowski, Rytter '95: constant space
- Gasienec, Kolpakov '04: real-time + sublinear space (extends GPR'95)
- • • more papers [Crochemore, Rytter '91,'95] [Crochemore '92] [...]
- Porat, Porat '09: randomized streaming, O(log m) space, no real-time
- Breslauer, Galil '10: randomized real-time streaming, O(log m) space

Our result

- Real-time constant-space string matching
  - O(1) words in addition to those for read-only X and T
  - O(1) worst-case time to answer after each text symbol

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- Real-time constant-space string matching deterministic
  - O(1) words in addition to those for read-only X and T
  - O(1) worst-case time to answer after each text symbol

Not to be confused with

- Real-time streaming string matching Montecarlo
  - O(log m) memory words (X and T cannot be kept)
  - O(1) worst-case time to answer after each text symbol

We propose a simple way to combine the two features

- Take a simple version of the constant-space Crochemore-Perrin (CP) algorithm

- Make CP also real-time by running two instances simultaneously

Simple version of the Crochemore-Perrin (CP) algorithm

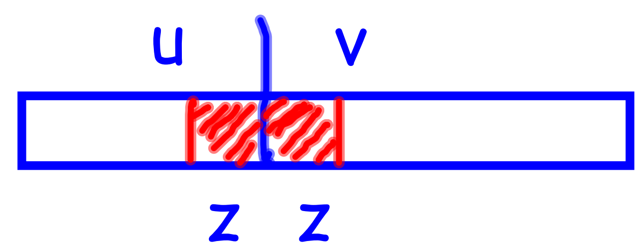
Consider a non-empty prefix-suffix factorization X = u v

The local period is the shortest z such that

z is suffix of u or vice versa

and

z is a prefix of v or vice versa



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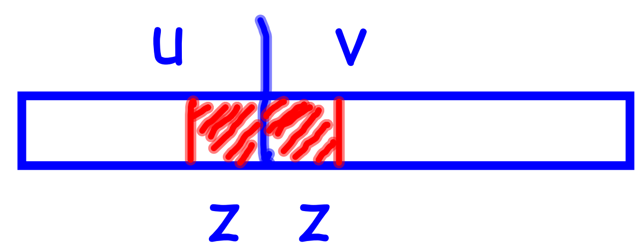

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Example: X = abaaaba

X = u v

a baaaba	ab aaaba	aba aaba
ba ba	aaab aaab	a a
z		

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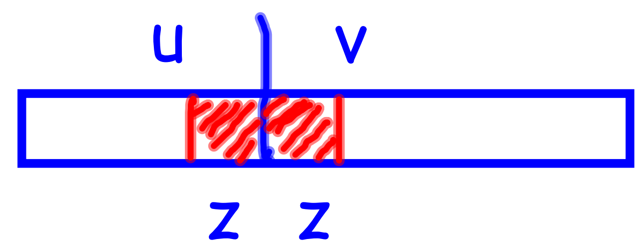

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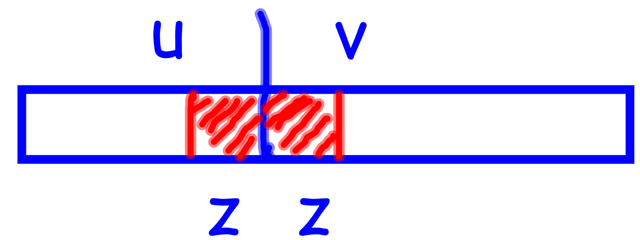

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Simple version of the Crochemore-Perrin (CP) algorithm

Consider a non-empty prefix-suffix factorization $X = u\, v$

The local period is the shortest z such that

z is suffix of u or vice versa

and

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$\mu(u,v) \equiv$ length of the local period

Critical factorization if $\mu(u,v) = \pi(X)$ [len. of the period of X]

Example:

X = u v

a baaaba	ab aaaba	aba aaba
ba ba	aaab aaab	a a

z

Example:

X = u v

a baaaba

ab aaaba

aba aaba

ba ba

aaab aaab

a a

z

Example:

```
 a  baaaba        X = u  v            aba  aaba
ba  ba              ab  aaaba           a  a
                  aaab  aaab
                         z
```

critical!

the period is "abaa"

Example:

a baaaba

ba ba

ab aaaba

aaab aaab

X = u v

aba aaba

a a

z

Example:

a baaaba   ab aaaba   aba aaba

ba ba   aaab aaab   a a

Critical Factorization Theorem (Cesari and Vincent):

Among $\pi(X) - 1$ consecutive factorizations:

at least one is a critical factorization

Example:

```
 a  baaaba        ab  aaaba        aba  aaba
ba  ba          aaab  aaab           a  a
```

Critical Factorization Theorem (Cesari and Vincent):

Among $\pi(X) - 1$ consecutive factorizations:

at least one is a critical factorization

There always exists a critical factorization $X = u\ v$ such that $|u| < \pi(X)$

Crochemore-Perrin (CP) Algorithm:

Take such a critical factorization of the pattern $X = u\ v$

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Back fill: match u left-to-right with the current aligned portion of the text [originally right-to-left]

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Take such a critical factorization of the pattern $X = u\ v$

Forward scan: match v left-to-right with the current aligned portion of the text

Back fill: match u left-to-right with the current aligned portion of the text [originally right-to-left]

How to handle mismatches?

We propose a simple way to combine the two features

- Take a simple version of the constant-space Crochemore-Perrin (CP) algorithm

- Make CP also real-time by running two instances simultaneously

Basic Real-Time Algorithm

Interleave O(1) comparisons from the forward scan with O(1) comparisons from the back fill

X = ab aaaba critical factorization

abaaaba

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Interleave O(1) comparisons from the forward scan with O(1) comparisons from the back fill

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z

abaaaba

≠

abaabaaabaa

↑

mismatch

Basic Real-Time Algorithm

Interleave O(1) comparisons from the forward scan with O(1) comparisons from the back fill

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z

abaaaba

abaabaaabaa

|z|+1

abaaaba

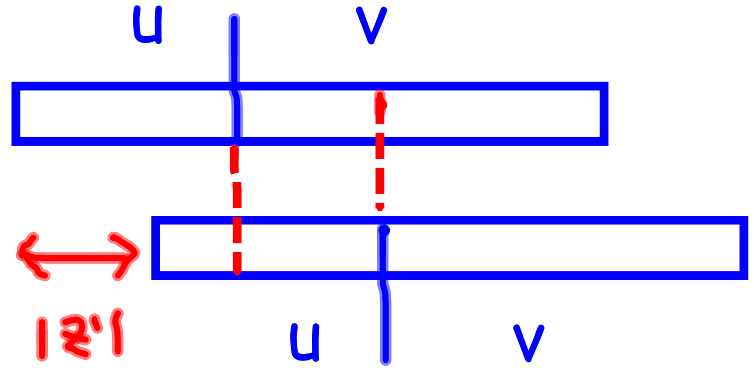
abaabaaabaa

shift by |z|+1 positions

(and charge the O(|z|+1) cost to the symbols in z in real time)

By contradiction, suppose there is a valid shift that is shorter...

... recall that |u| < π(X), the length of the period



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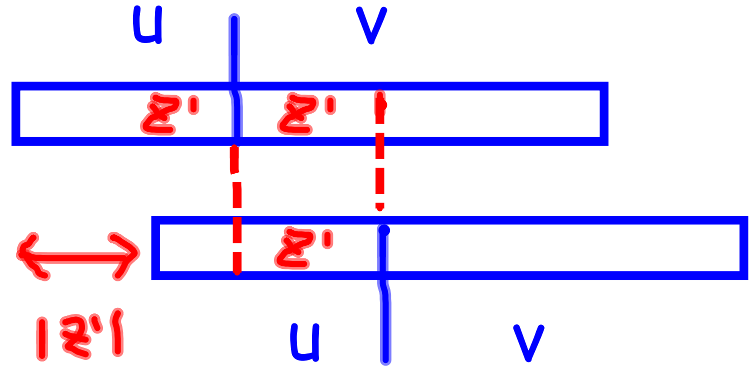

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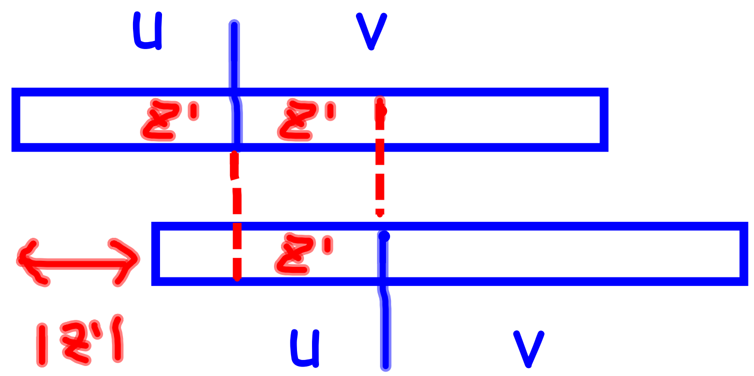

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Contradiction: a local period at u v that is shorter than $\pi(X)$!!

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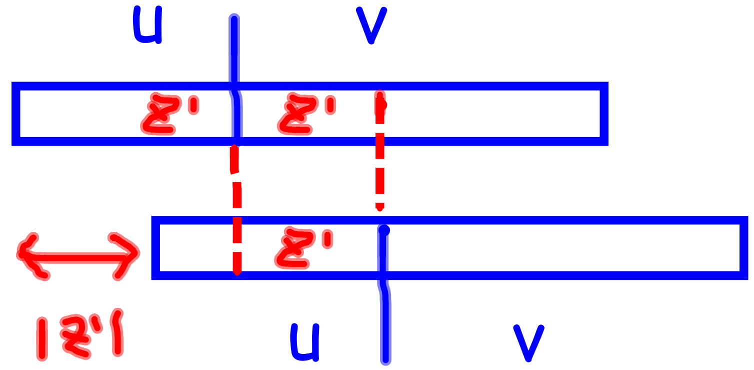

$|z'| < \pi(X)$

Contradiction: a local period at u v that is shorter than $\pi(X)$!!

It follows from the Crochemore-Perrin result [other case $|z'| \geq \pi(X)$ not displayed: periodicity rules out occurrences]

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Interleave O(1) comparisons from the forward scan with O(1) comparisons from the back fill

Output an occurrence when the forward scan terminates (and interrupt the back fill if needed)

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Let z be the matched prefix of v, where $X = u\,v$ is c.f.:

- if $z \neq v \Rightarrow$ shift by $|z|+1$ positions and reset z = empty
- if $z = v \Rightarrow$ shift by $\pi(X)$ positions and update z

Basic Real-Time Algorithm

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Output an occurrence when the forward scan terminates (and interrupt the back fill if needed)

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- if $z \neq v \Rightarrow$ shift by $|z|+1$ positions and reset z = empty
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Total cost is $O(1)$ worst-case per symbol:
the algorithm is real-time

Q: What if $|u| > |v|$?

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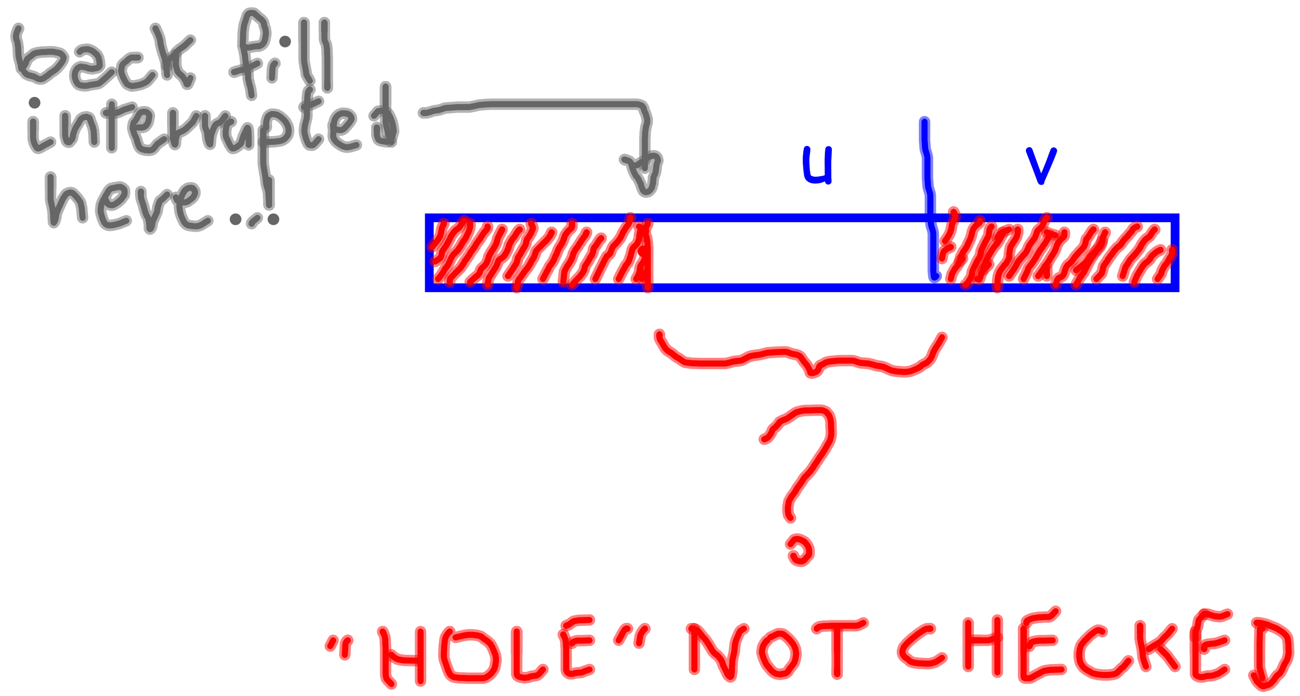

Real-Time Variation of CP

Consider a 3-way non-empty factorizaton $X = u\ v\ w$ such that

$X = (uv)\ w$ is a critical factorization with $|uv| \leq |w|$

OR

$X = (uv)\ w$ is a critical factorization, and
$X' = u\ (vv')$ is a critical factorization for a prefix X' of X
with $|u| \leq |vv'|$

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we focus on this...

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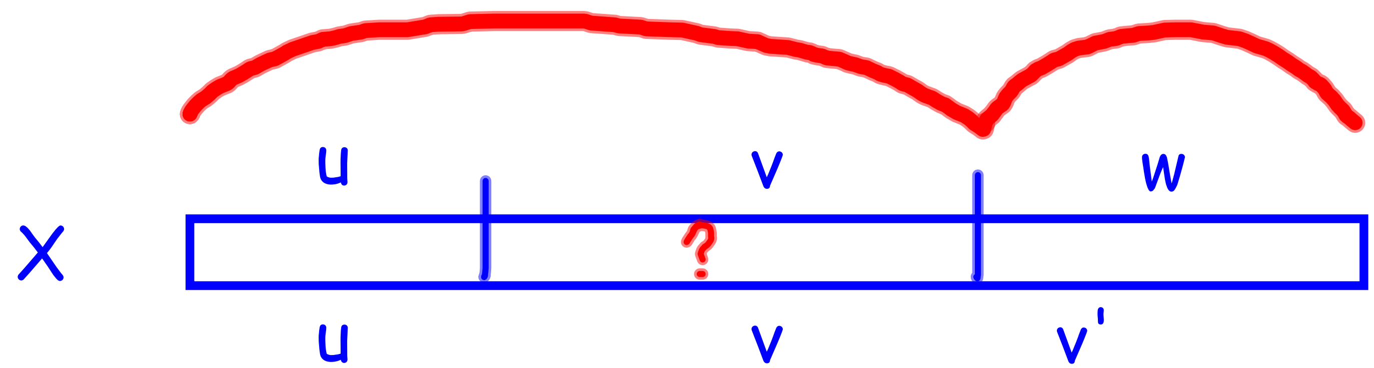

Recall we may leave a "hole" to the left of w:
this hole has to be covered by X'...

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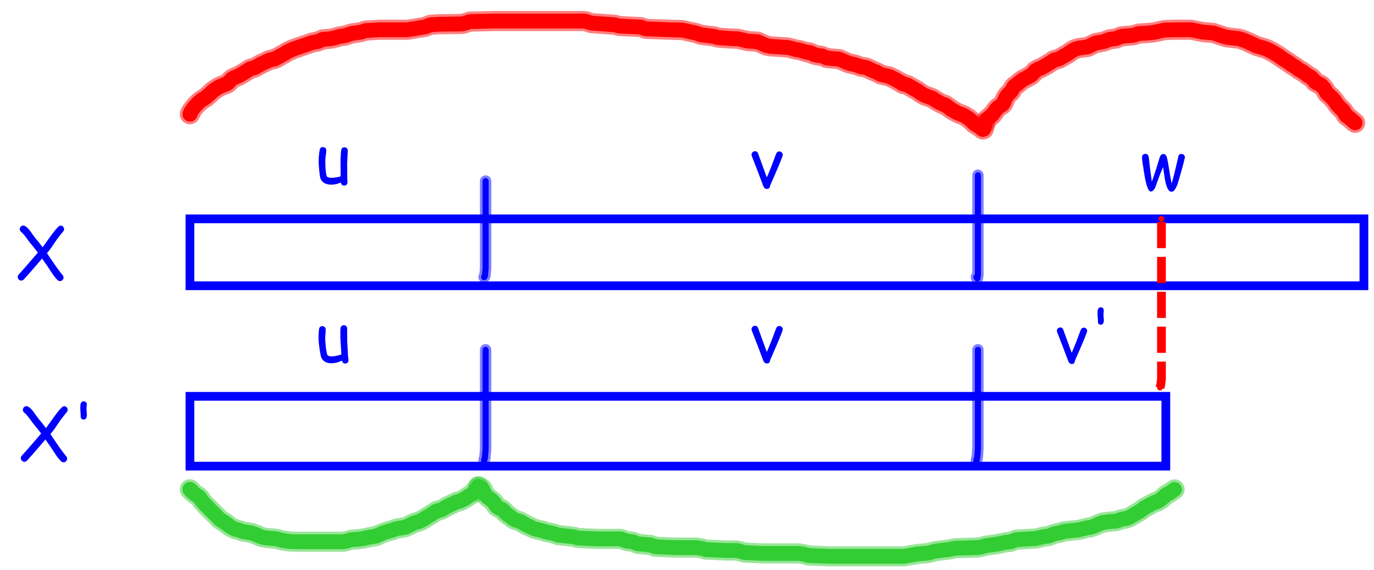

Note that X' is entirely matched since $|u| \le |vv'|$

Real-Time Variation of the CP Algorithm

Interleave O(1) steps of two instances of the Basic Real-Time Algorithms, one looking for X and the other for X', aligned with |X|-|X'| positions apart.

Real-Time Variation of the CP Algorithm

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Total cost is O(1) worst-case per symbol: the algorithm is real-time and reports correctly all the occurrences

Simple pseudocode

Pattern preprocessing

GOAL:

Find the desired 3-way non-empty factorizaton $X = u\ v\ w$
and the length of the periods of X and X'

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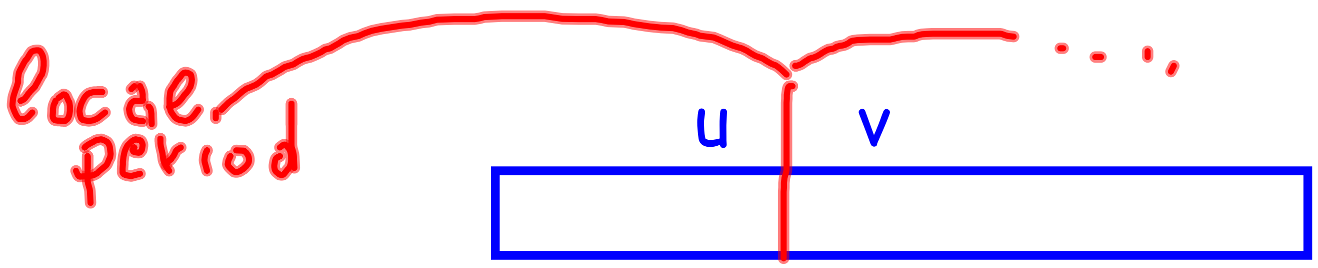
Some more definitions...

A factorization u v is left-external if $|u| \leq \mu(u,v)$ for non-empty u, v



Define L(X) = { u v : X = u v is left-external }

L(X) non-empty because of the Critical Factorization Theorem

Pattern preprocessing

Let $X = u_1 w$ be the first critical factorization in $L(X)$

HINT: use CP preprocessing on the prefixes of X

Lemma: $u\, v \in L(X) \Rightarrow$ prefix $X' = u'\, v'$ s.t. $\mu(u', v') = \mu(u, v)$

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Lemma: $u\ v \in L(X) \Rightarrow$ prefix $X' = u'\ v'$ s.t. $\mu(u',v') = \mu(u,v)$

Compute CP critical factorization for $u_1 = u\ v$

where $|u| \leq \mu(u,v)$

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Compute CP critical factorization for $u_1 = u\ v$

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Extend u_1 by periodicity $\mu(u,vw) < |vw|$: set $X' = u\ (vv')$

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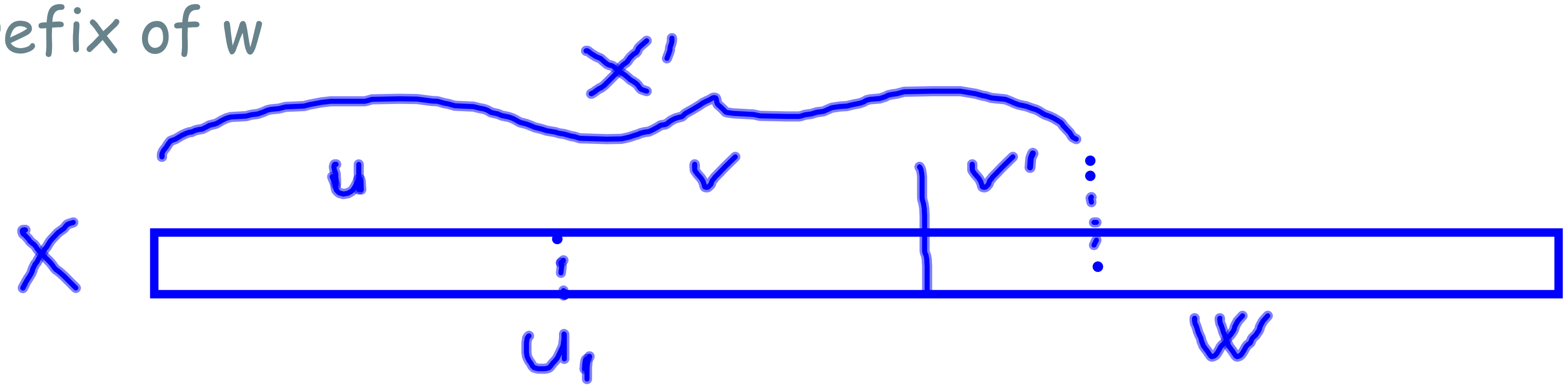

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It is $|u| \leq \mu(u, v) \leq \mu(u, vv') = \mu(u, vw) \leq |vv'|$

Q.E.D.

Questions?